Differential equation of S.H.M

If a particle undergoes SHM about a mean position then if 'F' is restoring force and 'x' is displacement from the mean position then from the condition of SHM we have

$$F \propto -x$$

$$\Rightarrow \boxed{F = -Kx} \quad \text{--- (1)}$$

where K is a constant of proportionality called force constant or spring factor and is expressed in Nm^{-1} in S.I. system

-ve sign shows that restoring force is directed towards mean position i.e opposite to the direction of displacement

From Newtons 2nd law of Motion.

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$$F = ma$$

or $$F = m\frac{dV}{dt}$$

$$F = m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

m = being mass of the body

a = acceleration

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or $\boxed{F = m \frac{d^2x}{dt^2}}$ —— (2)

using This in equation (1) we have

$$m \frac{d^2x}{dt^2} = -Kx$$

or $m \frac{d^2x}{dt^2} + Kx = 0$

Dividing both sides by 'm'

$$\frac{d^2x}{dt^2} + \left(\frac{K}{m}\right)x = 0 \quad \text{—— (3)}$$

Let us put $\frac{K}{m} = \omega^2$ —— (4)

then eqⁿ (3) becomes

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{—— (5)}$$

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eqⁿ (3) or eq (5) represents the differential eqⁿ. of S.H.M